

2025

MATHEMATICS

(Major)

Paper : MAT0500104

(Multivariate Calculus)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks
for the questions

1
93
X 14

372
93 X

1302

1. Answer the following as directed :

1×8=8

(a) If

$$f(x, y) = \frac{1}{\sqrt{9 - x^2 - y^2}}$$

then find the domain of it.

(b) Define level curve of a function $f(x, y)$ at a constant c .

(c) Find f_x if $f(x, y) = x^2 \sin(3x + y^3)$.

(2)

(d) If $f(x, y) = \tan xy$, then df is

- (i) $y \sec^2 xy dy + x \sec^2 xy dx$
- (ii) $y \sec^2 x dy dx + x \sec^2 xy dy$
- (iii) $\sec^2 xy dx + \sec^2 xy dy$
- (iv) $\sec^2 x dy + \sec^2 y dx$

(Choose the correct answer)

(e) ~~If $P_0(x_0, y_0)$ is a critical point of $f(x, y)$ and f has continuous 2nd-order partial derivatives in a disk centered at (x_0, y_0) and $D = f_{xx} f_{yy} - f_{xy}^2$, then a relative maximum occurs at P_0 , if~~

- (i) $D(x_0, y_0) > 0$ and $f_{yy}(x_0, y_0) > 0$
- (ii) $D(x_0, y_0) > 0$ and $f_{yy}(x_0, y_0) < 0$
- (iii) $D(x_0, y_0) < 0$ and $f_{yy}(x_0, y_0) > 0$
- (iv) $D(x_0, y_0) < 0$ and $f_{yy}(x_0, y_0) < 0$

(Choose the correct answer)

(f) ~~Find the Jacobian of the transformation from Cartesian coordinate to polar coordinate system.~~

(g) ~~State when a vector field C is said to be conservative.~~

(h) ~~Find curl of the vector field~~

$$\vec{V}(x, y, z) = u(x, y, z) \hat{i} + v(x, y, z) \hat{j} + w(x, y, z) \hat{k}$$

(Continued)

(3)

2. Answer any six of the following questions :

$$2 \times 6 = 12$$

(a) Show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^4 + y^2}$$

does not exist.

(b) Compute the slope of the tangent line to the graph of $f(x, y) = x \ln(x + y^2)$ at the point $P_0(e, 0, -e)$ in the direction parallel to the XZ-plane.

(c) Find the critical point of

$$f(x, y) = (x - 2)^2 + (y - 3)^4$$

and classify them.

(d) Find $\iint_R x \sin xy dA$ where

$$R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq 1\}$$

(e) If $\vec{F} = \vec{\nabla}f$ is the gradient of the function

$$f(x, y, z) = -\frac{1}{x^2 + y^2 + z^2}$$

then by using fundamental theorem of line integral, find the work done by \vec{F} along the smooth curve C joining $(1, 0, 0)$ to $(0, 0, 2)$ that not passes through the origin..

(4)

(f) Evaluate $\int_4^7 \int_1^2 \int_0^3 x^2 y^2 z^2 dx dy dz$.

(g) Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9, 0 \leq z \leq 5$.

(h) Find $\operatorname{div} \vec{F}$, given that $\vec{F} = \nabla f$ where $f(x, y, z) = x^2 y z^3$.

(i) State Green's theorem.

(j) Evaluate $\int_C (x + y) dx$ if C be the curve represented by $x = 2t, y = 3t^2, 0 \leq t \leq 1$.

3. Answer any four of the following questions :

5×4=20

(a) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s

$$\text{if } w = x + 2y + z^2, x = \frac{r}{s}, \quad y = r^2 \ln s,$$

$$z = 2r.$$

$$\Rightarrow N \frac{\partial}{\partial s} N^{-1}$$

$$\Rightarrow N (-1) N^{-2}$$

(b) Show that

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} = -N \frac{N}{s^2}$$

is continuous at every point except at the origin.

$$\frac{\partial}{\partial s} \left(-\frac{N}{s} \right)$$

$$N \frac{\partial}{\partial s} \left(-\frac{1}{s} \right)$$

(Continued)

$$\frac{\partial}{\partial M} \left(\frac{1}{s} \right)$$

$$(x - x_0) + (y - y_0) + \cancel{(z - z_0)}$$

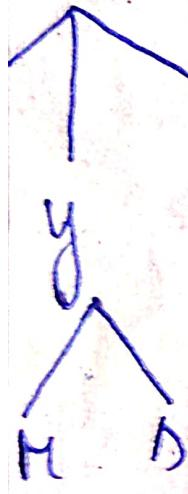
(5)

$z - z_0$

(c)

Find the equation of the tangent plane and the normal line at $P_0(1, -1, 2)$ on the surface S is given by $x^2y + y^2z + z^2x = 5$.

W



(d)

Find all critical points on the graph $f(x, y) = 8x^3 - 24xy + y^3$ and use the second partial test to classify as a relative extrema or a saddle point.

(e)

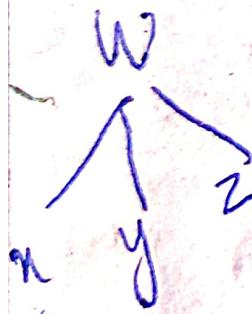
Find the work done by the force field $\vec{F} = (x^2 + y^2)\hat{i} + (x + y)\hat{j}$ as an object moves counter clockwise along the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and then back to $(1, 0)$ along the x -axis.

(f)

Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where

$$(z - z_0) =$$

$$\vec{F} = (y^2 - z^2)\hat{i} + 2yz\hat{j} - x^2\hat{k}$$



and C is curve defined parametrically by $x = t^2, y = 2t, z = t$ for $0 \leq t \leq 1$.

(g)

Find the mass of a lamina of density $\delta(x, y, z) = z$ in the shape of the hemisphere $z = (a^2 - x^2 - y^2)^{\frac{1}{2}}$.

$$8 \times 3\pi^2 \\ 24\pi^2$$

(h)

Evaluate the integral

$$\frac{M}{A}$$

$$\vec{F} \cdot d\vec{r}$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$$

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$$M \frac{\partial}{\partial A} A^{-1} \frac{\partial}{\partial A} \left(\frac{M}{A} \right)$$

$$M (-1) A^{-2}$$

$$\frac{M}{2A^2} - M$$

(Turn Over)

(6)

4. Answer any two of the following questions :

10×2=20

(a) (i) Suppose the function f is differentiable at the point P_0 and that the gradient at P_0 satisfies $\vec{\nabla}f_0 \neq 0$. Then show that $\vec{\nabla}f$ is orthogonal to the level surface of f through P_0 . 5

(ii) Let $f(x, y, z) = xyz$ and let \vec{u} be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = \hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of \vec{u} . 5

(b) Show that the vector field

$$\vec{F} = 2x(y^2 + z^2)\hat{i} + 2x^2y\hat{j} + 3x^2z^2\hat{k}$$

is conservative. Also find its scalar potentials. 4+6=10

(c) Evaluate

$$\iiint_D (x^2 + y^2 + z^2) dx dy dz$$

where D denotes the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = a, a > 0$. 10

(d) Use the method of Lagrange's multiplier to minimize

$$f(x, y) = 16 - x^2 - y^2$$

subject to $x + 2y = 6$. 10

(7)

(e) (i) State divergence theorem. By using divergence theorem, evaluate $\iint_S \vec{F} \cdot \vec{N} dS$ where

$$\vec{F} = x^2 \hat{i} + xy \hat{j} + x^3 y^3 \hat{k}$$

and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes with outward unit normal vector \vec{N} . 1+5=6

(ii) Find the area of the region D bounded above the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$. 4

$$x^2 + 2y = 2y$$

$$x^2 + y^2 = 0$$

$$\frac{\partial}{\partial y} [2\pi(y^2 + z^2)]$$

$$2\pi \frac{\partial}{\partial y} [y^2 + z^2]$$

$$2\pi \frac{\partial y}{\partial y}$$